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AN

INTRODUCTION TO THE Doctrine of *Fluxions*,

And DEFENCE of the *MATHEMATICIANS* AGAINST THE OBJECTIONS of the Author of the *ANALYST*, ſo far as they are deſigned to affect their general Methods of Reaſoning.

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LONDON:
Printed for J. NOON, at the *White-Hart*, near
Mercers-Chapel, in *Cheapſide*. MDCCXXXVI.

1736

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PREFACE.

I *HAVE long ago thought that the firſt principles and rules of the method of Fluxions ſtood in need of a more full and diſtinct explanation and proof, than what they had received either from their firſt incomparable author, or any of his followers; and therefore was not at all diſpleaſed to find the method itſelf oppoſed with ſo much warmth by the ingenious author of the* Analyſt; *and had it been his only deſign to bring this point to a fair iſſue, whether a demonſtration by the method of Fluxions be truly ſcientific or not, I ſhould have heartily applauded his conduct, and have thought he deſerved the thanks even of the Mathematicians themſelves. But the invidious light in which he has put this debate, by repreſenting it as of conſequence to the intereſts of religion, is, I think, truly unjuſtifiable, as well as highly imprudent. Among all wiſe and fair inquirers, 'tis beyond all contradiction plain, that religion can be no ways affected by the truth or falſhood of the doctrine of Fluxions. And tho' prejudiced minds may be variouſly affected by it, yet it is not eaſy to be conceived what advantage this debate is likely to give to the cauſe of religion and virtue in general even among* them. *Whereas it is eaſy to gueſs of what diſſervice our author's repreſentation of a controverſy in which* *religion has no manner of concern, may be towards raiſing and inflaming the paſſions of weak men on both ſides of the queſtion: And I wiſh he had been pleaſed coolly to conſider beforehand of what conſequence the reſult of this diſpute is likely to be to the cauſe of religion, among thoſe for whoſe conviction his* Analyſt *is chiefly deſigned. If he ſhould not be able to make out his point, will not the blind followers of the Infidel Mathematicians be more confirmed in their errors than they were before? Will they not be more prejudiced againſt religion, and eſtabliſhed in their eſteem and veneration of their maſters by a weak and fruitleſs attempt to depreſs their characters; and by finding that a zeal for it has occaſioned ſo ſtrong an attempt to wound the reputation of Sir* Iſaac Newton *as a cautious and fair reaſoner? And on the other hand, if our author ſhould carry his point, and his proofs ſhould be allowed, that the doctrine of Fluxions is an incomprehenſible myſtery, and that the moſt accurate Mathematicians have, one after another, impoſed upon themſelves in the moſt egregious manner, by falſe and inconcluſive reaſonings, what conſequences can we ſuppoſe that ſuch perſons will draw from theſe premiſes? Our Author indeed would have them only from hence make this one concluſion, That their maſters, the Mathematicians, are not to be depended upon when they ſpeak againſt religion. But I believe it can't in reaſon be expected that they ſhould ſtop here.*

If ſuch men as Dr. Barrow, *Dr.* Clarke, *&c. and the incomparable Sir* Iſaac Newton, *were capable of imagining that they ſaw with the greateſt clearneſs and perſpicuity, where they had nothing but abſolute and incomprehenſible darkneſs before them, what concluſion will perſons, uſed to take their opinions from authority, be likely to make from*

from theſe premiſes, but that all pretences to knowledge in religion, and every thing elſe, are only confidence and preſumption?

If they are taught that it is inconſiſtent for a perſon to reject the myſteries of religion, and yet believe the myſtery of Fluxions, will they not know how to draw the oppoſite concluſion themſelves, that it is inconſiſtent to reject the doctrine of Fluxions becauſe myſterious, and yet receive the myſteries of religion? And * *when they are taught to think that a perſon may be* juſtly *ſaid to have faith, becauſe they give into what they can neither demonſtrate nor* conceive; *if this give them a mean opinion of the Mathematicians, 'tis odds if it don't give them a mean opinion of faith itſelf. I am ſure 'tis a very ſtrange account of that which may juſtly be called faith: For without clear notions no man can believe any more than demonſtrate.*

Conſidering theſe things, I can't help thinking it was highly wrong to bring religion at all into this controverſy, which may inflame the diſpute, but can hardly do any real ſervice: Of which, to me, it is a very ſtrong preſumption, that every thing urged by the author of the Analyſt *againſt Infidels in general, would have ſounded full as well in the mouth of a Papiſt, if urged againſt thoſe Mathematicians that don't believe the doctrine of Tranſubſtantiation, as it would have been peculiarly in character for ſuch a one to have made his chief attack upon a great enemy of all ſuperſtition and tyranny, and an hearty friend to the reaſonable religion of Chriſtians and Proteſtants. But enough of this. I ſhall now conſider my ſubject as ſtript of all relation it has to religion, and merely as a matter of human ſcience, and endeavour to ſhew that the method of Fluxions is founded upon clear and ſubſtantial principles.*

SECT.

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* Defence of Freethinking, *p.* 62.

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SECT. I.

It cannot be doubted but that Sir *Iſaac Newton* well underſtood the doctrine of which he was the original inventor; and his proofs of it are very far from being fallacious and deceitful, or their force hard to be underſtood by thoſe that are uſed to theſe kinds of ſubjects.

But it is alſo very plain, that the queſtion, which is the main diſpute between our author and his adverſaries, whether Mathematicians take the notion and certainty of the method of Fluxions implicitly from him or not, does not depend upon our being able to defend the exact accuracy of the demonſtrations he has made uſe of, and the propriety of every phraſe by which he has explained his notions upon this head. He always ſeems to have ſtudied conciſeneſs of expreſſion, and to depend on the good ſenſe and judgment of his reader. And on this account ſome of his demonſtrations are not the moſt full and compleat that might be given, and muſt remain obſcure to thoſe who have no genius for the mathematical ſcience, and can't find out thoſe ſteps in a demonſtration, which a writer often omits in confidence of the ſagacity of his reader. In my opinion this is in ſome meaſure the caſe with reſpect to his proofs of the firſt principles of Fluxions, and therefore I don't wonder perſons differ in their ſentiments about them. But it is truly provoking to find that the greateſt genius that ever appeared in the philoſophical world, and one whom the lovers of knowledge muſt always think of with reſpect and gratitude, ſhould be repreſented contrary to his known character, as craftily impoſing on the world, in confidence of his own authority, and the obſcurity of his ſubject. And therefore I would hope that the author of the *Analyſt* did not deſign that ſevere reflection his words ſeem to carry with them, when he ſays, "Such reaſoning as this, nothing but the "obſcurity of the ſubject could have *encouraged* or *induced* the great author of the fluxionary method *to have put upon* his followers; "and nothing but an *implicit* deference to his "authority could have moved them to admit." To ſuſpect Sir *Iſaac Newton* of the mean deſign of ſeeking reputation among the ignorant, by venting unintelligible notions, and defending them by artful and cunning ſophiſtry, is what I think no man is capable of doing. And therefore if the author of the *Analyſt* does not think fit, for his own reputation, to revoke or explain the ſentence juſt mentioned, it needs not a particular confutation. Nor do I propoſe particularly to follow him in all the objections he has made againſt Sir *Iſaac*'s notions and demonſtrations, being of opinion that the beſt way of anſwering him is to aſſiſt perſons in underſtanding the ſubject itſelf; for if any one can do this, he will eaſily ſee there is little weight in what he has ſaid againſt it. However, as I go along, I ſhall endeavour to obviate any thing that I think may create a difficulty; but my main

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view is to ſettle the firſt principles on which the doctrine of Fluxions depends, and then to ſhew that, by juſt reaſoning from them, the rules for finding the Fluxions of equations, as delivered by Sir *Iſaac*, do truly follow.

THE notion of Fluxions was originally gained by the obſervation of quantities being deſcribed by a continual motion; and the method of Fluxions was deſigned to do theſe two things. *1ſt*, From the magnitude of a quantity continually changing being given, to find out the rate or velocity according to which the quantity itſelf continually increaſes or decreaſes. And, *2dly*, From this latter continually given, to find the former.

WHERE you ſee the main thing taken for granted is this

POSTULATE.

Quantities may be ſuppoſed as continually changing, ſo as every diſtinct inſtant of time to be different from what they were before.

ILLUSTRATION.

SUCH quantities are the following, Time from a given hour, the diſtance of a body from a plane to or from which it moves, the amount of money lent out at intereſt, &c. However, it is not the buſineſs of the Mathematician to diſpute whether quantities do in fact ever vary in the manner that is ſuppoſed, but only whether the notion of their ſo doing be intelligible; which being allowed, he has a right to take it for granted, and then to ſee what deductions he can make from that ſuppoſition. It is not the buſineſs of a Mathematician to ſhow that a ſtrait line or circle can be drawn, but he tells you what he means by theſe; and if you underſtand him, you may proceed farther with him; and it wou'd not be to the purpoſe to object that there is no ſuch thing in nature as a true ſtrait line or perfect circle, for this is none of his concern: he is not inquiring how things are in matter of fact, but ſuppoſing things to be in a certain way, what are the conſequences to be deduced from them; and all that is to be demanded of him is, that his ſuppoſitions are intelligible, and his inferences juſt from the ſuppoſitions he makes. In the caſe before us, Whether quantities in fact do ever vary in the manner before explained, or not, is nothing to the purpoſe, but whether the notion of their ſo doing be intelligible; and that it is ſo is plain, becauſe tho' this ſhould not be fact in any caſe, yet in a great many it ſeems to us ſo to be. When a ſtone falls to the ground, the line it deſcribes ſeems continually to increaſe; nor can I avoid the ſame ſentiment when I deſcribe a circle or any other line upon a paper. And it may as well be pretended that the letters I now write conſiſt of a number of diſtinct and ſeparate dots, and no continued lines, as that each of them does not ſeem to be formed by a continued motion. Now if I really think I ſee quantities formed by a continued motion, it is plain that this ſort of increaſe is not unintelligible, and therefore may be ſuppoſed by the Mathematician. The reader perhaps may here think that I intend to obviate an objection that even the author of the *Analyſt* himſelf would never make; but I muſt own my ſuſpicion of the contrary, becauſe I think he can't allow of ſuch ſort of increaſe, without giving up his cauſe, and allowing me a right to make this farther

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POSTULATE II.

The notion of FLUXIONS *is intelligible.*

FOR if quantities may increaſe or decreaſe ſo as to be different every diſtinct inſtant of time from what they were before, it will follow that they muſt change at a certain rate either fixed or variable, it being impoſſible that a man ſhou'd conceive of a quantity continually changing, without knowing that it muſt either alter at the ſame rate always, or elſe ſometimes faſter, and at other times ſlower; *i. e.* he can't do this without knowing what Sir *Iſaac Newton* means when he defines the Fluxion of a quantity to be the velocity or ſwiftneſs with which the quantity changes its magnitude: And the Fluxion of a quantity cannot be an unintelligible notion, when it neceſſarily ariſes from a plain and eaſy ſuppoſition. Nay, I am very well ſatisfied that our author himſelf muſt have a notion both of a firſt and ſecond Fluxion, if he at all underſtands himſelf, when he ſuppoſes *a line deſcribed by the motion of a point continually accelerated.* At leaſt I am ſure 'tis as hard for me to conceive of the motion of a point, and the acceleration of that motion, as to form an idea of a firſt and ſecond Fluxion.

SECT. II.

DEFINITIONS.

1. A FLOWING quantity is one that continually increaſes or decreaſes, and in ſuch a manner that ſome time is requiſite to make any increment or decrement.

2. THE Fluxion of a flowing quantity is its rate or ſwiftneſs of increaſe or decreaſe.

3. THE change of a flowing quantity is the difference between the flowing quantity itſelf, and its value at a particular inſtant of time.

4. THE time in which a change is made, is the time the flowing quantity takes to alter from a prior to a ſubſequent value, whoſe difference is the change.

5. THAT change is ſaid to vaniſh at a given inſtant, which is the difference between the flowing quantity before that inſtant and its value then; and that change is ſaid to begin to ariſe at a given inſtant, which is the difference between the flowing quantity after that inſtant, and its value at that inſtant.

ILLUSTRATION.

LET y be a quantity continually increaſing, A its value at a given inſtant; then $A-y$ will repreſent its change [taken from that inſtant] ſo long as y is leſs than A; and this change will vaniſh when y becomes equal to A; and by the continued increaſe of y, there will then begin to ariſe another change which is ever afterwards equal to $y - A$.

Def. 6. If there be two permanent quantities A and B, and two other flowing Quantities a and b, and the ratio of a to b be always, during a given time, that of the ſum or difference of the firſt permanent quantity A, and another flowing quantity x to the ſum or difference of the ſecond permanent quantity B, and another flowing quantity y, and at the end of the given time all the flowing quantities vaniſh; then the ratio of the permanent quantities A and B, is the laſt or ultimate ratio of the vaniſhing quantities a and b; which I thus expreſs, ult. $a : b :: A : B$. *i. e.* if $a : b :: A \mp x : B \mp y$ always, during the time T, and at the end of that time, a, b, x, y all vaniſh; then ult. $a : b :: A : B$.

7. If other things being ſuppoſed the ſame, a, b, x, y, be each equal to nothing at the beginning of the time T, then the ratio of A to B is the firſt ratio of the naſcent quantities a and b; which I thus expreſs, $\text{p}^{\text{mo}}\ a : b :: A : B$.

Remark. These two definitions are in effect the ſame with thoſe given by Sir *Iſaac Newton*, and can't be diſputed; for whether a and b, properly ſpeaking, have any proportion as they ariſe, or vaniſh, yet A and B have; and that I am at liberty to call by what name I pleaſe.

Coroll. 1. The ultimate ratio of two vaniſhing quantities, is a determinate ratio; *i. e.* if ult. $a : b :: A : B$, and ult. $a : b :: A : N$. B is $= N$. For ſince ult. $a : b :: A : B$; therefore, by *Def.* 6. $a : b :: A \mp x : B \mp y$ during a given time T, at the end of which the flowing quantities a, b, x, y, all vaniſh; and by the ſame

Defin. $a:b::A\mp v:N\mp z$, during a given time t, at the end of which the flowing quantities a, b, v, z, all vaniſh; and the end of t and T coincide, becauſe that is the inſtant when a and b vaniſh. Taking therefore the ſhorter of theſe times t, during that $A\mp x:B\mp y::A\mp v:N\mp z$. Now becauſe x, y, z, v, change only as flowing quantities, and vaniſh together at the end of the time t, before the end of the time t they will be all leſs than any aſſignable quantity D [for ſuppoſe any of them never leſs than D before that inſtant, then a decrement as large as D muſt be made inſtantaneouſly, in order to its vaniſhing at that inſtant, which is contrary to the ſuppoſition of its changing only as a flowing quantity.] But ſince x, y, v, z, may be as ſmall as you pleaſe, and yet the analogy $A\mp x:B\mp y::A\mp v:N\mp z$ be preſerved, it is plain that $A:B::A:N$; and that therefore $B=N$. Q. E. D.

Coroll. 2. IN like manner it may be proved that the firſt ratio of two ariſing quantities is a given ratio.

AXIOM I.

THE ſum and difference of two flowing, and the fourth proportional to three flowing quantities; and therefore a quantity any how made up of given and flowing quantities, muſt be itſelf a flowing or permanent quantity; *i. e.* ſome time is requiſite to its receiving any increment or decrement, becauſe a change in it muſt imply a change at leaſt in ſome one of them.

Coroll. 3. IF $a:b::A\mp x:B$ during the time T; then if, at the end of the time T, the flowing quantities a, b, x vaniſh, ult. $a:b::A:B$; and if, at the beginning of T, they are equal to nothing, p^{mo} $a:b::A:B$. For becauſe $a:b::A\mp x:B$, therefore $a:b::A+a\mp x:B+b$ during the time T, and a and b, and $a\mp x$ are flowing quantities all vaniſhing together, or all ariſing together; and therefore, by *Defin.* 6 and 7, in the former caſe ult. $a:b::A:B$, and in the latter p^{mo} $a:b::A:B$.

Coroll. 4. IF $a:b::A:B$ always, a and b vaniſhing or ariſing together, then ult. $a:b::A:B$; or p^{mo} $a:b::A:B$. This is proved as the foregoing.

Coroll. 5. IF the ratio of two quantities continually decreaſing approach continually nearer and nearer to a given ratio, and by ſo doing at length come nearer to it than by any aſſignable difference, that given ratio is the laſt ratio of the flowing quantities when they vaniſh. Let the flowing quantities be a and b, the given ratio $A:B$. Then ſuppoſing $a:b::A:B\mp x$ always, as a and b decreaſe, x will alſo continually decreaſe in the ſame manner as a flowing quantity, by *Ax.* 1. and x may be as ſmall as you pleaſe, becauſe the ratio of a to b may approach nearer to that of A to B, than by any aſſignable difference. And therefore when a and b vaniſh, x muſt vaniſh with them; [for if it then be equal to D, it never before was leſs than D, ſince it was continually decreaſing; whereas before it was ſmaller than any aſſignable quantity] and therefore, by *Coroll.* 3. ult. $a:b::A:B$.

Coroll. 6. IF two flowing quantities a and b ariſe from nothing at a given inſtant, and the nearer the time be taken to that of their riſe, ſo much the nearer is their ratio to that of A to B; and the time may be taken ſo ſmall, that the ratio of a to b ſhall differ from that of A to B, leſs than by any aſſignable difference, in this caſe, p^{mo} $a : b :: A : B$. The Proof of this is ſimilar to that of the foregoing.

Remark. IN theſe two laſt corollaries you have Sir *Iſaac Newton*'s deſcription of the ultimate ratio of vaniſhing quantities, and the * prime ratio of naſcent or ariſing ones, which is in effect the ſame with that given in *Defin.* 6, 7. But I choſe to make this little alteration for the greater expedition in practice. And it is to be noted, that tho' Sir *Iſaac*, to explain what he means by the ultimate ratio of vaniſhing quantities, deſcribes it as in *Coroll.* 4. yet in practice he ſeems evidently enough to make uſe of one ſimilar to that of *Defin.* 6.

Coroll. 7. IF ult. $a : b :: A : B$, then ult. $a + b : b :: A + B : B$. For ſince ult. $a : b :: A : B$, by *Defin.* 6. a time T may be aſſumed; during which $a : b :: A \mp x : B \mp y$; and at the end of which the flowing quantities a, b, x, y, all vaniſh; and therefore alſo during that time, $a + b : b :: A + B \mp x \mp y : B \mp y$; and at the end of that time alſo, the four quantities $a + b$, b, $x \mp y$, and y, which are flowing quantities, by *Axiom* 1. vaniſh: wherefore, by *Defin.* 6. ult. $a + b : b :: A + B : B$.

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* We have not, as I know of, any direct definition of theſe prime ratio's, but we are left to form it from that moſt accurate definition of the ultimate ratio's of vaniſhing quantities; which we have at the latter end of *Schol. Lemm.* II. *Princ.* and which is ſo plain, that I wonder how our author could help underſtanding it; which had he done, I am apt to think that all his *Analyſt* ſays concerning the proportion of quantities vaniſhing with the quantities themſelves, had never been heard: For according to this definition, we are not obliged to conſider the laſt ratio as ever ſubſiſting between the vaniſhing quantities themſelves. But between other quantities it may ſubſiſt, not only after the vaniſhing quantities are quite deſtroyed, but before when they are as large as you pleaſe. And the reaſon we conſider quantities as decreaſing continually till they vaniſh, is not in order to make, but to find out, this laſt ratio. Sir *Iſaac Newton* does indeed ſay that this laſt ratio is the ratio with which the quantities themſelves vaniſh; but whether he herein ſpeaks with the utmoſt propriety or not, is a mere nicety on which nothing at all depends.

Coroll. 8. If ult. $a : b :: A : B$, then ult. $a - b : b :: A - B : B$; A being greater than B.

Coroll. 9. If ult. $a : b :: A : B$, and ult. $b : d :: B : D$; then alſo by equality, ult. $a : d :: A : D$.

Remark. These two laſt corollaries are demonſtrated by the like contrivance as the ſixth; and the like propoſitions are true, and in like manner to be proved concerning prime ratio's.

Axiom 2. A quantity flows uniformly, or with a permanent Fluxion, when the changes made in it are always proportional to the times in which they are made.

Axiom 3. The Fluxions of quantities uniformly flowing, are always in the proportion of their ſynchronal changes; and therefore, becauſe this is a given ratio (by *Coroll.* 4. *Defin.* 5 and 6.) in their ultimate ratio when they vaniſh, or their prime ratio when they ariſe.

Axiom 4. If thro' any time two quantities be generated, or changes be made in two quantities, one with an uniform Fluxion or rate of increaſe or decreaſe, and the other with a Fluxion continually increaſing; then the ratio of the quantity generated, or the change made by the uniform or permanent Fluxion to the quantity generated, or the change made by the continually increaſing Fluxion, will be always leſs than the ratio of the Fluxion of the former is to the Fluxion of the latter at the beginning of the time, and greater than the ratio which the Fluxions are in to one another at the end of the time.

Axiom 5. BUT if, other things being ſuppoſed the ſame, the latter Fluxion continually decreaſe, the ratio of the quantity generated, or the change made by the uniform or permanent Fluxion to the quantity generated, or the change made by the continually decreaſing Fluxion, will be always greater than the ratio of the Fluxion of the former is to the Fluxion of the latter at the beginning of the time, and leſs than the ratio of the Fluxion of the former to the Fluxion of the latter at the end of the time. Both theſe axioms are plain conſequences from this obvious truth, that if any thing increaſe with an accelerated velocity or ſwiftneſs, the increaſe made in a given time will be greater than if it had all along increaſed with the ſame ſwiftneſs it did at the beginning, and leſs than if it had all along increaſed with the ſwiftneſs it did at the end of it; and that the contrary will happen if the velocity be continually retarded. And the reader will pleaſe to take notice, that as the foregoing axioms are the ſole principles on which the doctrine of Fluxions is built, ſo they were generally allowed, before this doctrine was ever thought of, in relation to the velocities of moving bodies, and the ſpaces deſcribed by their motion. The only thing new in this caſe is, that the idea of velocity is render'd more general, and, in order to prevent ambiguity, the word Fluxion has been ſubſtituted in the room of it: but ſtill the ſame general notion is preſerved; for as velocity, at leaſt in its more uſual ſenſe, ſignifies the degree of quickneſs with which a body changes its ſituation in reſpect of ſpace, ſo the Fluxion of a quantity ſignifies the degree of quickneſs with which the quantity changes its magnitude. Why then is the notion of a Fluxion harder to be underſtood than that of a velocity? Why may not we make uſe of the ſame axioms in relation to one, which all the world allows in relation to the other? Surely ſome reaſon ought to be aſſigned of the difference between the two caſes; or it is not fair to make thoſe things peculiar objections againſt the notion of Fluxions, which equally affect the notion of velocities. And I am apt to think that the reader is miſtaken, if he imagines that Sir *Iſaac Newton* is peculiarly attacked in the objections of the *Analyſt* on this head, and not the common ſenſe of mankind both before and ſince his time. And what good reaſon can be aſſigned why the common notions of all mankind are repreſented as the peculiar blunders of Sir *Iſaac* and his followers, it is not eaſy to determine. As to the ſecond, *&c.* Fluxions, there may, I allow, ſeem to be ſome peculiar difficulty in them to perſons that are not uſed to theſe ſubjects; but it ſeems to me great weakneſs to imagine that there can be any peculiar objections againſt them, becauſe tho' the farther we go in the orders of Fluxions, our ideas become more and more complex, yet no new ſimple ideas are admitted. And when our author aſſerts, that in order to conceive of a ſecond Fluxion, we muſt conceive of the velocity of a velocity, and that this is nonſenſe; he plainly appeals to the ſound and not the ſenſe of words. When velocity is conſidered only as an affection of motion, the velocity of a velocity is nonſenſe. But then this is not the notion of a ſecond Fluxion; and if by inlarging the idea of velocity, you make it ſynonymous to the word *Fluxion*, then the velocity of a velocity, however oddly it may ſound, is nothing but plain common ſenſe. For the degree of quickneſs with which any quantity increaſes, may vary to greater or leſs with any degree of quickneſs imaginable; and this laſt is called the ſecond Fluxion of the quantity. Now if the author of the *Analyſt* can ſhew the abſurdity of this notion, he is welcome ſo to do; but let him not firſt dreſs it up in an ambiguous phraſe, and then think to confute it by laughing at his own way of underſtanding that phraſe.

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PROP. I.

If there be two flowing quantities, one of which flows with a permanent Fluxion, and the other with a continually increaſing one, then the Fluxions of theſe quantities, at any given inſtant of time, will be in the laſt ratio of their ſynchronal changes vaniſhing at that inſtant, and in the prime ratio of their ſynchronal changes which begin to ariſe at that inſtant.

1. LET the given inſtant be the fix'd end of the variable time *t*, the determinate values of the two flowing quantities at the given inſtant *A* and *B*, their Fluxions then $\dot{y}$ and $\dot{z}$; the flowing quantities themſelves y and z; the ſynchronal changes made in the time *t* in the quantities y, z, and the Fluxion of z, which continually increaſes, and I ſuppoſe now only as a flowing quantity, whilſt the Fluxion of y remains always the ſame, call reſpectively $\acute{y}$, $\acute{z}$, and $\acute{\dot{z}}$. Then ſince $\dot{z}$ is the Fluxion of z at the given inſtant, or the end of *t*, and $\acute{\dot{z}}$ the change made in it during the time *t*, and this Fluxion continually increaſes; therefore $\dot{z} - \acute{\dot{z}}$ will always expreſs the Fluxion of z at the beginning of *t*, whether *t* be longer or ſhorter; and conſequently, by *Axiom* 4. the ratio of $\acute{y}$ to $\acute{z}$ will be

always leſs than that of $\dot{y}$ to $\dot{z}-\overset{\prime\cdot}{z}$, and greater than that of $\dot{y}$ to $\dot{z}$. If therefore $\overset{\prime}{y}:\overset{\prime}{z}::\dot{y}:\dot{z}-x$ always, x will be always leſs than $\overset{\prime\cdot}{z}$. Now as t continually decreaſes, it is plain, by *Axiom* 1. that the quantities $\overset{\prime}{y}$, $\overset{\prime}{z}$, x, $\overset{\prime\cdot}{z}$ being always made up of given and flowing quantities, can themſelves only vary as flowing quantities, and as ſuch they muſt vaniſh, when t vaniſhes that is at the given inſtant: [$\overset{\prime}{y}$ muſt then vaniſh, being always equal to the difference between A and y; and A and y are equal at the given inſtant: for the ſame reaſon $\overset{\prime}{z}$ and $\overset{\prime\cdot}{z}$ vaniſh at the given inſtant, and x alſo muſt vaniſh, it being all along only a part of $\overset{\prime\cdot}{z}$] Wherefore ſince $\overset{\prime}{y}:\overset{\prime}{z}::\dot{y}:\dot{z}-x$ always, and $\overset{\prime}{y}$, $\overset{\prime}{z}$, and x varying only as flowing quantities, vaniſh at the given inſtant, (by *Cor.* 3. *Def.* 6.) ult. $\overset{\prime}{y}:\overset{\prime}{z}::\dot{y}:\dot{z}$.

2. AGAIN; let T denote any time that begins at the given inſtant, $\dot{y}$ and $\dot{\mathfrak{z}}$ the Fluxions of the quantities at the beginning of the time T; $\overset{\prime}{y}$, $\overset{\prime}{\mathfrak{z}}$, and $\overset{\prime\cdot}{\mathfrak{z}}$, ſynchronal changes of the quantities and of the Fluxion that increaſes, made in the time T, and other things being ſuppoſed as before. Then by a like reaſoning as the foregoing, it will appear, from *Axiom* 5, that if $\overset{\prime}{y}:\overset{\prime}{\mathfrak{z}}::\dot{y}:\dot{\mathfrak{z}}+x$, that x is always leſs than $\overset{\prime\cdot}{\mathfrak{z}}$; and by *Axiom* 1. and *remark* at the end of *Coroll.* 9. that $\text{p}^{\text{mo}}\ \overset{\prime}{y}:\overset{\prime}{\mathfrak{z}}::\dot{y}:\dot{\mathfrak{z}}$.

3. IF the Fluxion of *z* at the beginning of the time *T*, receives an inſtantaneous increment, which is not impoſſible, nor contrary to any thing ſuppoſed in the propoſition, then *z* is to be looked upon as having two Fluxions at the given inſtant, one of which is that it would have had if no ſuch alteration had been in it; and the other is that it would have had if the ſame alteration had been made in it before the given inſtant, and not then. And taking the times *t* and *T* ſo ſhort, that during both there be no other inſtantaneous alteration made in the Fluxion of *z*, beſides that at the given inſtant; and following the ſteps of the foregoing demonſtrations, you will find that the ultimate ratio of the changes of *y* and *z* vaniſhing at the given inſtant, is the ſame as the ratio of their Fluxions, on the ſuppoſition that no alteration had been made in the Fluxion of *z* at the given inſtant; and that the prime ratio of the changes of *y* and *z* beginning to ariſe at the given inſtant, is the ſame as the ratio of their Fluxions, on the ſuppoſition that the alteration in the Fluxion of *z* had been made before the given inſtant: all which is manifeſt, ſince on the former ſuppoſition, the Fluxion of *z* thro' the time *t*, and on the latter thro' the time *T*, varies only as a flowing quantity. Wherefore, *&c.*

W. W. D.

PROP. II.

IF two quantities flow, one uniformly, and the other with a decreaſing Fluxion, or with a Fluxion increaſing to a given inſtant, and then decreaſing continually, or *vice verſâ*; and even tho' this Fluxion ſhould not always before or after the given inſtant increaſe, or always decreaſe, but for ſome time only immediately before it, ſhou'd continue in the ſame ſtate of increaſe or decreaſe, and ſo likewiſe immediately after it; yet ſtill the Fluxions of theſe quantities will be in the laſt ratio of their ſynchronal changes vaniſhing at the given inſtant, and in the prime ratio of their ſynchronal changes which begin to ariſe at that inſtant; *i. e.* this is true in whatever manner one of theſe quantities flow. All which is to be made out by following the ſteps of the demonſtration of the foregoing propoſition, having aſſumed your times t and T ſo ſhort, that during either of them there be no alteration of the Fluxion from a ſtate of increaſe to that of decreaſe, or *vice verſâ*.

PROP. III.

IF two quantities flow any how, their Fluxions, at a given inſtant of time, will be in the laſt ratio of their ſynchronal changes vaniſhing at that inſtant, and in the firſt ratio of ſynchronal changes then beginning to ariſe.

1. LET the given inſtant be the end of the time t, the flowing quantities z and x, an uniformly flowing quantity y; changes made in theſe quantities in the time t, $\overset{\prime}{z}$, $\overset{\prime}{x}$, $\overset{\prime}{y}$; their Fluxions at the end of t, $\dot{z}$, $\dot{x}$, $\dot{y}$. Then (by Prop. 2.) ult. $\overset{\prime}{z} : \overset{\prime}{y} :: \dot{z} : \dot{y}$, and by the ſame ult. $\overset{\prime}{y} : \overset{\prime}{x} :: \dot{y} : \dot{x}$; wherefore, by equality, ult. $\overset{\prime}{z} : \overset{\prime}{x} :: \dot{z} : \dot{x}$.

2. LET the given inſtant be the beginning of T, the changes made in the time T, $\overset{\prime}{Z}$, $\overset{\prime}{X}$, $\overset{\prime}{Y}$, and their Fluxions at the beginning of T, $\dot{Z}$, $\dot{X}$, $\dot{Y}$.

Then alſo (by Prop. 2.) $\text{p}^{\text{mo}}\, \overset{\prime}{\dot{Z}} : \overset{\prime}{\dot{Y}} : : \dot{Z} : \dot{Y}$, and $\text{p}^{\text{mo}}\, \overset{\prime}{\dot{Y}} : \overset{\prime}{\dot{X}} : : \dot{Y} : \dot{X}$; and therefore, as before, $\text{p}^{\text{mo}}\, \overset{\prime}{\dot{Z}} : \overset{\prime}{\dot{X}} : : \dot{Z} : \dot{X}$. W. W. D.

LEMMA I.

IF a determinate ratio be never greater than the greateſt, nor leſs than the leaſt of two variable ratio's, it will not be greater than the greateſt, nor leſs than the leaſt of thoſe ratio's when they are prime or ultimate; *i. e.* if $A : B$ is never greater than $x : y$, nor leſs than $v : z$, it will be a mean alſo between the ultimate or prime ratio of x to y, and v to z; or of the ultimate ratio of x to y, and the prime ratio of v to z, or *vice verſâ*. All which is plain; for, by an evident conſequence from the definitions of prime and ultimate ratio's, the quantities v, z, x, y, may be ſo ſmall, as that their ratio's ſhall differ leſs from their prime and ultimate ratio's than by any aſſignable difference; and therefore if, $x : y$ being the greater of the two ratio's, $A : B$ ſhould be greater than the ultimate ratio of x to y, let the difference between them be D. Then becauſe $x : y$ may be nearer to the ultimate ratio of x to y, than by the difference D, the ratio of $A : B$ may be greater than that of x to y; but it is ſuppoſed to be never greater; which things are inconſiſtent: and therefore $A : B$ is never greater than the ultimate ratio of x to y. And in like manner every thing elſe aſſerted in this *Lemma* may be proved.

Coroll. HENCE if the ultimate ratio of v to z, and the prime ratio of x to y be equal; and the ratio of A to B be never leſs than that of x to y, and never greater than that of v to z; then the ultimate ratio of v to z, and the prime ratio of x to y, will be each equal to that of A to B.

PROP. IV.

SUPPOSING that at the time you ſeek the Fluxions, there be no inſtantaneous change made in them; *i. e.* ſuppoſing the Fluxions you ſeek to be themſelves flowing quantities, or permanent ones, then if you can take ſynchronal increments or decrements of each, made partly before and partly after a given inſtant, in ſuch a manner, that however ſmall they ſhall always be in a given ratio, that ratio will be the ratio of the Fluxions of the quantities at the given inſtant; *i. e.* if the flowing quantities be z and x; and if whilſt z changes from $A - y$ to $A - y + a$, x changes from $B - v$ to $B - v + b$, a being greater than y, and b than v; and the ratio of a to b be a given ratio, then the ratio $a : b$ is the ratio of the Fluxions of z and x at the inſtant that $z = A$ and $x = B$, A and B being permanent quantities: For calling the Fluxions of z and x at that time $\dot{z}$ and $\dot{x}$, ſince v and y are* ſynchronal changes of z and x vaniſhing at the given inſtant, ult. $y : v :: \dot{z} : \dot{x}$; and becauſe $a - y$ and $b - v$ are ſynchronal changes of the ſame quantities ariſing at the given inſtant, and there are no inſtantaneous

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* See *Defin.* 5.

changes made in the Fluxions, therefore p^{mo} $a - y : b - v :: \dot{z} : \dot{x}$; but the ratio of a to b, as is evident, is never greater than the greateſt, nor leſs than the leaſt of the ratio's $y : v$, and $a - y : b - v$; and therefore (by *Coroll. Lemm.* preced.) the ratio $a : b$, which is a given ratio, is equal to $\dot{z} : \dot{x}$, which is the ult. ratio of $y : v$, and the prime ratio of $a - y : b - v$. W.W.D.

Remark 1. It muſt be owned that this propoſition affords only a more indirect way of finding the Fluxions of quantities, and has this diſadvantage above the foregoing, that it not only cannot be applied where there is an inſtantaneous change made in the Fluxions; (it then giving only a mean between the greateſt and leaſt Fluxion of the quantity whoſe Fluxion you ſeek) but alſo when the Fluxion of the quantity you ſeek, compared with a permanent Fluxion, changes from an increaſing to a decreaſing one, or *vice verſâ*. However I thought proper to take notice of it, becauſe in particular caſes it affords the moſt elegant manner of demonſtrating the propoſitions of Fluxions: and the way of demonſtrating from this propoſition, has at leaſt ſome ſimilitude to that which Sir *Iſaac Newton* uſes in *Lemm.* 2. Lib. II. *Princ.*

Remark 2. The obſervation I made at the end of Prop. 1. That the Fluxion found out from the laſt ratio's and firſt ratio's may poſſibly be different at the ſame inſtant of time, cannot create any difficulty in practice, nor require that we ſhould inveſtigate them both ways,* becauſe if the Fluxions of any quantities as found out one way, appear to be flowing quantities, they muſt be ſo in reality, and therefore are always the ſame when found out either way. In what follows, therefore, I ſhall only conſider Fluxions as found from the ultimate ratio's of the vaniſhing changes, but the reader will eaſily ſee that exactly the ſame concluſions would reſult juſt in the ſame manner, by arguing from the prime ratio's of the changes that begin to ariſe at any inſtant.

PROP. V.

THE Fluxion of the ſum of two quantities is, at the end of any given time, or always, equal to the ſum of the Fluxions of each, if they both increaſe or decreaſe together. Let the two flowing quantities be z and y, their ſum, by *Axiom* 1, is a flowing quantity, which call s; then is $z + y = s$ always; and calling the Fluxions of theſe three quantities at the end of any given time $\dot{z}$, $\dot{y}$, $\dot{s}$, and their ſynchronal changes vaniſhing then $\acute{z}$, $\acute{y}$, and $\acute{s}$. Then becauſe z and y increaſe or decreaſe together, $\acute{z} + \acute{y} = \acute{s}$ always; therefore ult. $\acute{s} : \acute{z} + \acute{y} :: 1 : 1$; but (by Prop. 3.) ult. $\acute{z} : \acute{y} :: \dot{z} : \dot{y}$, and therefore

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* *i. e.* By conſidering the changes made in the flowing quantities as the differences between their values at the given inſtant and their former values, which differences vaniſh at the given inſtant; and then conſidering them as the differences between their values at the given inſtant, and their ſubſequent values, which differences begin to ariſe at the given inſtant. See *Defin.* 3 and 5.

ult. $\acute{z} + \acute{y} : \acute{y} :: \dot{z} + \dot{y} : \dot{y}$; and therefore from the firſt and third analogies it follows that ult. $\acute{y} : \acute{s} :: \dot{y} : \dot{z} + \dot{y}$; but (by Prop. 3.) ult. $\acute{y} : \acute{s} :: \dot{y} : \dot{s}$; and therefore becauſe the ultimate ratio of two vaniſhing quantities is a given ratio $\dot{y} : \dot{z} + \dot{y} :: \dot{y} : \dot{s}$, and $\dot{z} + \dot{y} = \dot{s}$; and conſequently the Fluxion of *s*, the ſum of two flowing quantities increaſing or decreaſing together, is always equal to the ſum of the Fluxions of each. W. W. D.

PROP. VI.

IF an equation involve ever ſo many flowing quantities, turn them all on one ſide, and then inſtead of each quantity putting its Fluxion, you will have the equation expreſſing the relation of the Fluxions when all the quantities flow one way; but if not, change the ſign before the Fluxions of thoſe quantities which decreaſe, and this will give you the equation of the Fluxions.

1. SUPPOSE the quantities to flow all one way, and that the Fluxions of *y*, *z*, *&c.* are $\dot{y}$, $\dot{z}$, *&c.* then if $z - y + x = v + r - s$ always, it will follow that $\dot{z} - \dot{y} + \dot{x} - \dot{v} - \dot{r} + \dot{s} = 0$. For from the firſt equation $z + x + s = v + r + y$ always; and taking $S = z + x$, $\Sigma = S + s$, and $R = v + r$, and $\omega = R + y$ always, 'tis plain that S, Σ, R, ω, which are flowing quantities, by *Axiom* 1. muſt flow the ſame way with *z*, *y*, *&c.* and therefore (by

Prop. 5.) $\dot S = \dot z + \dot x$, $\dot\Sigma = \dot S + \dot s$, $\dot R = \dot v + \dot r$, and $\dot\omega = \dot R + \dot y$; and becauſe $\Sigma = \omega$ always, therefore $\dot\Sigma = \dot\omega$, and $\dot R + \dot y = \dot S + \dot s$; that is, $\dot v + \dot r + \dot y = \dot z + \dot x + \dot s$, or $\dot z - \dot y + \dot x - \dot v - \dot r + \dot s = 0$. And the ſame way this part of the propoſition is evident, if there had been ever ſo many flowing quantities in the equation.

2. SUPPOSE ſome of the quantities to decreaſe whilſt others increaſe, and let the equation be $Z - Y + v - x = R + r + T$ always; I ſay then, that if the great letters denote increaſing quantities, and the ſmall ones decreaſing ones, that $\dot Z - \dot Y - \dot v + \dot X - \dot R - \dot T + \dot r = 0$. For taking $V = 2D - v$, and $X = D - x$, and $P = D - r$ always, D being a determinate quantity large enough, it is evident that whilſt v, x, r decreaſe, V, X, P muſt increaſe, and that $Y + R + T + V = Z + X + P$ always. Wherefore by the foregoing part of this propoſition, $\dot Y + \dot R + \dot T + \dot V - \dot Z - \dot X - \dot P = 0$. But becauſe $V = 2D - v$ always, the ſynchronal changes of V and v are always equal, ſince D is a ſtanding quantity; and therefore $\dot V = \dot v$. And in like manner $\dot X = \dot x$, and $\dot P = \dot r$. Putting therefore $\dot v$, $\dot x$, $\dot r$ in the places of $\dot V$, $\dot X$, $\dot P$, it will follow that $\dot Z - \dot Y - \dot v + \dot x - \dot R - \dot T + \dot r = 0$. And in the ſame way the juſtneſs of the

rule might have been demonſtrated (as is evident) had there been ever ſo many more quantities in the equation.

Coroll. THE ſame rule holds, tho' beſides the flowing quantities there be determinate ones in the equation, ſuppoſing their Fluxions $= 0$, as is too evident to need any demonſtration.

N. B. IN practice this change of the ſign is neglected as uſeleſs, but then we conſider the Fluxions of decreaſing quantities as negative, which comes to the ſame thing.

PROP. VII.

LET A be a determinate quantity, z and x flowing ones, their Fluxions $\dot{z}$ and $\dot{x}$; I ſay then, that if $Az = x$ always, $A\dot{z}$ will be $= \dot{x}$ always: for $z : x :: 1 : A$; and therefore the ſynchronal changes, and conſequently the Fluxions of z and x, will be always in the conſtant ratio of 1 to A; *i. e.* $\dot{z} : \dot{x} :: 1 : A$, and conſequently $A\dot{z} = \dot{x}$ always. W.W.D.

PROP. VIII.

LET x and y be two flowing quantities whoſe Fluxions are $\dot{x}$ and $\dot{y}$ always; I ſay then, that if $x^2 = y$ always, that $2x\dot{x} = \dot{y}$ always. Let A be the value of x at a given inſtant, $\acute{x}$ and $\acute{y}$ ſynchronal changes of x and y vaniſhing at that inſtant, $\dot{X}$ and $\dot{Y}$ their Fluxions at the ſame inſtant;

ſtant; then becauſe $x^2 = y$ always, $2A\acute{x} \mp \acute{x}^2 = \acute{y}$ always, as is evident; and therefore $\acute{x} : \acute{y} :: 1 : 2A \mp \acute{x}$ always, and therefore ult. $\acute{x} : \acute{y} :: 1 : 2A$ (by *Coroll.* 3. *Defin.* 6.) but ult. $\acute{x} : \acute{y} :: \dot{X} : \dot{Y}$ (by Prop. 3.) $\dot{X}$ and $\dot{Y}$ being the Fluxions of x and y at the inſtant their ſynchronal changes $\acute{x}$ and $\acute{y}$ vaniſh; and therefore $\dot{X} : \dot{Y} :: 1 : 2A$, or $2A\dot{X} = \dot{Y}$. But at the given inſtant, when $\dot{x} = \dot{X}$, and $\dot{y} = \dot{Y}$, x is alſo $= A$, and therefore at that inſtant $2x\dot{x} = \dot{y}$. And becauſe the ſame may be proved in the ſame manner at any other inſtant of time, therefore $2x\dot{x} = \dot{y}$ always.

W. W. D.

Or thus:

WHILST x changes from $A - \frac{1}{2}\acute{x}$ to $A - \frac{1}{2}\acute{x} + \acute{x}$, x^2 or y changes from $A^2 - A\acute{x} + \frac{1}{4}\acute{x}^2$ to $A^2 - A\acute{x} + \frac{1}{4}\acute{x}^2 + 2A\acute{x}$; and therefore (by Prop. 4.) the ratio of $\acute{x}$ to $2A\acute{x}$, or 1 to $2A$, being a conſtant ratio, is the ratio of the Fluxions of x to x^2 or y at the inſtant that x becomes $= A$, and $x^2 = A^2$.

PROP.

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PROP. IX.

LET x, y, and z be three flowing quantities whoſe Fluxions are always $\dot{x}$, $\dot{y}$, and $\dot{z}$; I ſay then, that if $xy = z$ always, and x and y flow the ſame way, that $\dot{x}y + \dot{y}x = \dot{z}$. But if whilſt one of them increaſes the other decreaſes, that $\dot{z}$ is equal to $\dot{x}y - \dot{y}x$, or $\dot{y}x - \dot{x}y$, according as z flows the ſame way with x or y. Let the ſum of x and y be v, and its Fluxion $\dot{v}$; then becauſe $x + y = v$ always, $v^2 = x^2 + 2yx + y^2$, *i. e.* $v^2 = x^2 + y^2 + 2z$ always. And if x and y flow the ſame way, ſo it is plain muſt v, v^2, y^2, x^2, and $2z$ do. Taking therefore the Fluxions (by Prop. 6 and 8.) $\dot{x} + \dot{y} = \dot{v}$, and $2v\dot{v} = 2x\dot{x} + 2y\dot{y} + 2\dot{z}$ always; but $2v\dot{v} = \overline{2\dot{x} + 2\dot{y}}$, $\times\, \overline{x + y} = 2x\dot{x} + 2y\dot{y} + 2\dot{x}y + 2\dot{y}x$ always, and therefore $\dot{z} = \dot{x}y + \dot{y}x$ always.

AGAIN; if z and y flowing one way, x flows the contrary, aſſume a determinate quantity A large enough, and ſuppoſe $A - s = x$ always, the Fluxion of s being $\dot{s}$ always; then will $Ay - sy$ be $= xy$, and conſequently $Ay - sy = z$ always. But here it is manifeſt, becauſe $A - s = x$ always, that s flows the contrary way to x, and therefore the ſame way with y and z; and therefore that Ay, sy, and z, and s and y all flow the ſame way, and the contrary way to x. And therefore from the two preceding equations collecting the Fluxions by the foregoing propoſitions, and the part of this that has been proved, $A\dot{y} - \dot{s}y - \dot{y}s = \dot{z}$, and $\dot{s} = \dot{x}$; and inſtead of s and $\dot{s}$ in the former of theſe, putting their values $A - x$ and $\dot{x}$, $A\dot{y} - \dot{x}y - \dot{y}A + \dot{y}x = \dot{y}x - \dot{x}y = \dot{z}$. And laſtly, if z and x flow one way, and y the contrary, in the ſame manner you may prove that $\dot{x}y - \dot{y}x = \dot{z}$.

Coroll. HERE if the Fluxions of decreaſing quantities are conſidered as negative ones, then in all caſes the Fluxion of $xy = \dot{x}y + \dot{y}x$. And from hence the rule in Sir *Iſaac Newton*, from the equation of the Fluents to find the equation of the Fluxions, is eaſily proved.

SECT. III.

I HAVE now proved, I hope, beyond exception, that conſidering Fluxions as the velocities or rates with which quantities increaſe or decreaſe, that their proportions may be always found, if the laſt ratio of their ſynchronal changes vaniſhing at every given inſtant is known.

But I would here obſerve, that whatever falſe metaphyſics there may be in the notion of Fluxions conſidered as velocities, it does not at all affect the general method; as Sir *Iſaac Newton* himſelf has informed us, that inſtead of theſe Fluxions we may make uſe of any other finite quantities found out from the laſt ratio of the ſynchronal increments or decrements of Flowing Quantities; and therefore the Author of the *Analyſt* hardly acted the part of a fair adverſary, in making ſuch a pother about the notion of Fluxions as incomprehenſible, ſince if he had not been able to underſtand them, he might have made uſe of any other quantities he did underſtand in their ſtead. I don't ſay this as if I thought there were any difficulty in the notion of Fluxions thus conſidered; I am on the contrary very poſitive that no man can make any objection againſt a velocity of increaſe or decreaſe in general, that will not as ſtrongly lie againſt a velocity of motion. If quantities may increaſe faſter or ſlower, as well as bodies move faſter or ſlower, there is no greater impropriety in ſaying that one quantity increaſes with a greater velocity than another, than in ſaying that one body moves with a greater velocity than another. But tho' I think there is no difficulty in the notion of Fluxions conſider'd as velocities, yet in order to underſtand equations where Fluxions of different orders are jumbled together, it would be convenient to repreſent all Fluxions not as before, but as quantities of the ſame kind with their Fluents; and therefore I ſhould chuſe to do it, were I to write a treatiſe on this ſubject: And this may be done in the following manner. Take *Defin.* 1, and *Axiom* 2 for the two firſt definitions: Define alſo ultimate ratio as before, and then proceed thus: *Defin*, 4. The Fluxion of an uniformly flowing quantity is the change made in it in a certain given time. *Defin*. 5. The Fluxion of a quantity any how flowing at any given inſtant, is a quantity found out by taking it to the Fluxion of an uniformly flowing quantity in the ultimate proportion of thoſe ſynchronal changes which then vaniſh. And conſidering Fluxions in this light, all the uſes might be made of them as are done under the foregoing notion. But I go on to what the *Analyſt* gives me more occaſion to take notice of. I might have proved from what went before, that the Fluxion of x is to that of x^n, as 1 to nx^{n-1}. But for the ſake of juſtifying Sir *Iſaac Newton*, I ſhall now take his method.

LET x increaſe uniformly, and 'tis propoſed to find the Fluxion of x^n.

WHILST x by flowing becomes $x+o$, x^n will become $\overline{x+o}|^n$, that is, $x^n + nox^{n-1} + \frac{n^2-n}{2}o^2x^{n-2} +$ &c. and the ſynchronal augments o and $nox^{n-1} + \frac{n^2-n}{2}o^2x^{n-2} +$ &c. are to one another as $1 : nx^{n-1} + \frac{n^2-n}{2}ox^{n-2}$ + &c.

LET now theſe augments vaniſh, and their laſt ratio will be as $1 : nx^{n-1}$; and therefore the Fluxion of x is to the Fluxion x^n, as 1 is to nx^{n-1}. W. W. F.

THIS our author ſays is no fair and concluſive reaſoning, becauſe when we ſuppoſe the "increments to vaniſh, we muſt ſuppoſe their "proportions, their expreſſions, and every thing "elſe derived from the ſuppoſition of their exi-

“ ſtence to vaniſh with them.” To this I anſwer, that our author himſelf muſt needs know thus much, *viz.* That the leſſer the increment o is taken, the nearer the proportion of the increments of x and x^n will arrive to that of 1 to nx^{n-1}, and that by ſuppoſing the increment o continually to decreaſe, the ratio of theſe ſynchronal increments may be made to approach to it nearer than by any aſſignable difference, and can never come up with it before the time when the increments themſelves vaniſh. And no more nor leſs than this does Sir *Iſaac* mean (as he himſelf informs us) when he ſays, that the laſt ratio of the vaniſhing increments is that of 1 to nx^{n-1}. When therefore our author muſt own that to be true which Sir *Iſaac* intends, what ſignifies it to diſpute whether it be proper to ſpeak of the proportion of the increments as ſtill in being, when the quantities themſelves vaniſh or become $= 0$.

FOR tho’, ſtrictly ſpeaking, it ſhould be allowed that there is no laſt proportion of vaniſhing quantities, yet on this account no fair and candid reader wou’d find fault with Sir *Iſaac Newton*, for he has ſo plainly deſcribed the proportion he calls by this name, as ſufficiently to diſtinguiſh it from any other whatſoever: So that the amount of all objections againſt the juſtice of his method in finding out the laſt proportion of vaniſhing quantities, can ariſe to little more than this, that he has no right to call the proportion he finds out according to this method by that name, which ſure muſt be egregious trifling. However, as on this head our author ſeems to talk with more than uſual confidence of the advantage he has over his opponents, and gives us what he ſays is the * amount of S^r^ *Iſaac*'s reaſoning in a truly ridiculous light, it will be proper to ſee on whom the laugh ought to fall, for I am ſure ſomebody muſt here appear ſtrangely ridiculous. His arguings and illuſtrations founded on || the *Lemma* he propoſes as ſelf-evident, I think I have no occaſion to meddle with, becauſe I readily allow whatever conſequence he is pleaſed to draw from it, if it appears that Sir *Iſaac*, in order to find the laſt ratio's propoſed, was obliged to make two inconſiſtent ſuppoſitions. To confute which nothing more need be ſaid than barely to relate the ſuppoſitions he did make.

1. THEN he ſuppoſes that x by increaſing becomes $x+o$, and from hence he deduces the relation of the increments of x and x^n.

2. AGAIN; in order to find the laſt ratio of the increments vaniſhing, he † ſuppoſes o to decreaſe till it vaniſhes, or becomes equal to nothing. Beſides theſe he makes no other ſuppoſitions, and theſe are evidently no more inconſiſtent and contradictory, than to ſuppoſe a man ſhould firſt go up ſtairs, and then come down again. To ſuppoſe the increments to be ſomething and nothing at the ſame time, is contradictory; but to ſuppoſe them firſt to exiſt, and then to vaniſh, is perfectly conſiſtent; nor will the conſequences drawn from the ſuppoſition of their prior exiſtence, if juſt, be any ways affected by the ſuppoſition of their ſubſequent vaniſhing, becauſe the truth of the latter ſuppoſition no

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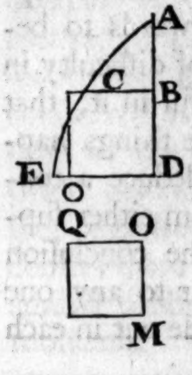
* *Analyſt*, p. 21. || Id. p. 20.

† That what is here made the ſecond ſuppoſition, is truly the meaning of *Evaneſcant jam augmenta illa* in Sir *Iſaac Newton*, is very plain from the manner in which he ſuppoſes quantities to vaniſh throughout the *Introduction to the Quadratures*.

ways contradicts the truth of the former. To make this more plain, conſider what is made out from each ſuppoſition: from the firſt that x has increaſed by o, this conſequence is drawn, that the proportion between the increments of x and x^n, *ſo long as they exiſt*, may be expreſſed by that of 1 to $nx^{n-1} + \frac{n^2-n}{2} o x^{n-2}$, *&c.* if o always expreſs the increment of x. And this conſequence is no ways affected by ſuppoſing o continually to decreaſe, and at length to vaniſh. But from this laſt ſuppoſition we may gather, that the leſſer o is, ſo much the nearer the ratio of 1 to nx^{n-1} comes to the ratio of the increments; and that by a continual diminution of o, it may come as near to it as you pleaſe, but can never equal it before o quite vaniſhes; and therefore this ratio, and no other whatſoever, agrees to the deſcription which Sir *Iſaac* has given of the ultimate ratio of the vaniſhing increments. His concluſion therefore comes out without the ſuppoſition of any thing inconſiſtent. And our Author, by his way of objecting, ſeems to make no diſtinction between two oppoſite things being done at the ſame time, and their being done at different ones; between a line's being drawn, and then rubbed out; and lines ſuppoſed to be drawn, and not drawn at all; as will appear ſtill farther by conſidering his objection againſt the way of finding out the ordinate of a curve from the area and abſciſſe being perpetually given (which is a problem analogous to that of finding the Fluxion from the Fluent perpetually given, and) which I ſhall now repreſent ſomewhat more diſtinctly than he has done, and then conſider his objection againſt it.



LET the area $ABC=A$, the abſciſſe $AB=b$, the ordinate $BC=k$, all which are ſtanding quantities as well as the ſquare $QM=M$, and its ſide $QO=q$; and let any other ordinate, with its correſpondent abſciſſe and adjoining area, as DE, AD, AED, be called reſpectively z, x, and V. And the nature of the curve is ſuch, that the area of the curve is to QM as the cube of the adjoining abſciſſe is to the cube of QO; that is, $A\times q^3=M\times b^3$, and $V\times q^3=M\times x^3$. From whence I find the area $BCED\times q^3=M\times\overline{x^3-b^3}$; and ſuppoſing $BCED=$ to the rectangle Bo, and calling Do, y, I find, inſtead of $BCED$, putting its value, and dividing by $x-b$, that $yq=x^2+xb+b^2$; which equation I ſee is true, let x be of any magnitude either greater or leſs than b; from whence I conclude it muſt be true alſo when $x=b$. And becauſe I find that y is always a mean between k and z, I conclude that when $x=b$, ſince then $z=k$, that then alſo $y=k$; and therefore that at that time the equation $yq=x^2+bx+b^2$ muſt degenerate into this, $kq=3b^2$, which gives me the relation of the ordinate and abſciſſe. W. W. F.

Now in this way of reaſoning, ſays our Author, there is a direct fallacy; becauſe, *firſt*, we are obliged to ſuppoſe that b and x are unequal, without which we could not proceed one ſtep; and, in the *ſecond* place, it is ſuppoſed that they are equal, which is a manifeſt inconſiſtency. But cannot ſo accurate a reaſoner as our Author diſtinguiſh between different times? To ſuppoſe x and b equal and unequal at the ſame time, would have been an inconſiſtency; but to ſuppoſe them firſt unequal, and afterwards to become equal, has not the ſhadow of difficulty in it. And there is nothing wonderful in it, that from the ſuppoſition of both theſe things happening one after another, I can deduce a concluſion which would not follow from either ſuppoſition ſingly taken; and that the concluſion I have drawn is juſt, will appear to any one that conſiders what is diſtinctly made out in each ſtep of the demonſtration.

BY taking x different from b, I find this equation, $yq = x^2 + xb + b^2$, and obſerve it holds true of whatever magnitude x be taken; the meaning of which may be thus expreſſed in words at length: There always is a line greater than one and leſs than the other of the two quantities k and z, ſo long as they remain different; which multiplied by q is always greater than one, and leſs than the other of the two quantities $3x^2$ and $3b^2$. Now this propoſition is always true whatever becomes of x afterwards, and from it I infer that at the time when $x = b$, and $k = z$, kq muſt alſo be equal to $3b^2$. The force of which inference depends upon this plain axiom, That if ſo long as two quantities remain different, a third be always greater than one and leſs than the other of them, as ſoon as by a continued increaſe or decreaſe any two of theſe quantities become equal, all three of them muſt be equal. The truth of which, were there any manner of occaſion for it, might eaſily be made out by reducing the contrary poſition to an abſurdity.

IN what has been ſaid, I have deſignedly taken notice only of thoſe objections of the ingenious author of the *Analyſt* which relate to the doctrine of Fluxions; as for the differential method of *Leibnitz*, I do not undertake its defence, becauſe I think the notion of Fluxions conſidered as velocities, or rather as finite quantities of the ſame kind with their Fluents, as before deſcribed, is much more eaſily conceivable, and frees from difficulty. Nay, I muſt confeſs there ſeems to me to be ſome objection againſt conſidering quantities as generated from moments. What moments, what the *principia jamjam naſcentia finitarum quantitatum*, are in themſelves, I own, I don't underſtand; I can't, I am ſure, eaſily conceive what a quantity is before it comes to be of ſome bigneſs or other; and therefore moments conſidered as parts of the quantities whoſe moments they are, or as really fixed and determinate quantities of any kind, are beyond my comprehenſion, nor do I indeed think that Sir *Iſaac Newton* himſelf did thus conſider them. But when he ſays, for inſtance, that the moment of A^2 is equal to the moment of A into twice A, his meaning is only this, that if the increment of A be continually diminiſhed, the proportion between that into A, and the correſpondent increment of A^2 will approach towards, and at length come nearer to, a ratio of equality than by any aſſignable difference. And thus underſtanding him, the proportion of moments is a phraſe eaſily intelligible, and his meaning very evident in every ſentence where he uſes it, altho', with me, you can't imagine what a moment in itſelf is. 'Tis true indeed, upon this ſuppoſition, it may not be agreeable to the exact accuracy of language, to ſpeak of moments as if they were real intelligible quantities; but I am ſure this is all that can juſtly be objected againſt it, and that there is no falſe logic or metaphyſics in it. Whether I have here ſpoke the ſenſe of S^r *Iſaac* himſelf, or not, I won't be poſitive, and muſt leave it to the judgment of the reader converſant in his works; but I think he has given us intimation ſufficiently plain, that we may at leaſt thus interpret him if we pleaſe, he being not at all ſolicitous under what notion we conſider moments, whilſt we take their proportions in that manner he directs us.

OUR Author may perhaps think himſelf not fairly uſed, that there has been no more notice taken of ſecond and third *&c.* Fluxions in this treatiſe, but it is to no purpoſe to defend theſe before the objections againſt the firſt are given up; and if the firſt are well underſtood, there is no manner of difficulty in the other, except what ariſes from this conſideration, that our ideas become ſomewhat more complex as we go on to the higher orders of Fluxions. However I ſhall mention one thing for the ſake of young beginners in this ſcience. Suppoſe $y^2 = x$, and y flows uniformly, then, according to the method of Fluxions, $2y\dot{y} = \dot{x}$, and $2\dot{y}^2 = \ddot{x}$, it may be inquired what is the meaning of this equation taking Fluxion in the notion of velocity, ſince upon that ſuppoſition firſt and ſecond Fluxions are quantities of different kinds, and therefore have no proportion to one another? To this I anſwer, that the only meaning of the equation is, that if you take the value of $2\dot{y}^2$ and $\ddot{x}$ at two different times, the $2\dot{y}^2$'s are in the ſame proportion to one another that the $\ddot{x}$'s are. And in like manner are equations always to be underſtood which involve quantities of different ſorts; they don't expreſs the proportion of heterogeneous quantities to one another, but of different values of homogeneous quantities among themſelves. And from hence our Author may, if he pleaſes, receive an anſwer to the latter part of his thirty-firſt query; an odd query, ſurely, to be made by one that pretends to anſwer Sir *Iſaac Newton*: But ſeveral of this kind are to be met with, which ſhew greater prejudice againſt the Mathematicians, than knowledge of the principles which they maintain. And I can't help obſerving, that tho' our Author profeſſes the utmoſt caution as to what he admits as true, and a concern for the utmoſt accuracy in reaſoning, yet neither the one nor the other appears when he is making his objections againſt the Mathematicians. Of this I ſhould not have taken any notice, if his deſign had only been to correct ſome miſtaken notions among the Mathematicians; but as his intention is evidently to run down their method of reaſoning, and to repreſent them as Bigots and Enthuſiaſts in their own ſcience, it is but a piece of juſtice to them to ſhew that their adverſary is as inaccurate in his reaſonings, and as incautious in his aſſertions, and in forming his accuſations, as any of them can be. Of this the reader may have already obſerved ſome inſtances, and I ſhall beg his patience whilſt I mention one or two more.

1. HE urges againſt the Mathematicians with great injuſtice, I think, that they imagine there may be a proportion between nothings, and yet he himſelf argues from this principle, *Anal.* p. 18. "Points are undoubtedly equal, as "having no more magnitude one than another, "a limit or point, as ſuch, having no magni-"tude."

2. EVERY reader of the *Analyſt* will obſerve what ſtreſs he lays on this maxim, Inaccurate premiſes cannot infer an accurate concluſion; as if nothing more was neceſſary than the belief of this principle to confute his adverſaries; whereas the inference he would draw from thence, that we therefore cannot, in any caſe, know the concluſion to be right, if we have argued from the ſuppoſition of the truth of what we know to be falſe, or don't know to be true, is what no rules of logic will juſtify: becauſe it is poſſible for me, by virtue of falſe or inaccurate premiſes, to gain a concluſion, and at the ſame time to know that the error in the premiſes will not cauſe any error in the concluſion. Thus in Algebra the ſuppoſition of a negative quantity, or a quantity leſs than nothing, is an abſurdity, but yet this is no hindrance to the evidence of thoſe concluſions that according to the rules of art are derived from it. And again, to bring an inſtance more to our preſent purpoſe, I'll allow that the ſuppoſition of an infiniteſimal is abſurd, and alſo that the ſuppoſition that an infiniteſimal added to a finite quantity does not increaſe its magnitude is abſurd, when an infiniteſimal is conſider'd as a fix'd determinate

quantity; and yet, I ſay, in deducing the value of the ſubtangent of a Parabola from theſe ſuppoſitions, I may be ſure that I make a right concluſion; becauſe thoſe quantities which I ſuppoſe to be infinitely ſmall, if they are not ſo will be very ſmall, and thoſe I ſuppoſe to be equal will be very nearly ſo, and therefore the error in the concluſion can be but very ſmall; *i. e.* the ſubtangent will be equal to twice the abſciſſe very nearly. But I am certain that this cannot poſſibly be demonſtrated concerning any finite and determinate quantities, unleſs they are truly and exactly equal, if you don't fix upon ſome determined proportion or difference as the ſtandard of what is nearly equal, but leave every perſon at liberty to chuſe what he will. And from this principle I would obſerve, that the concluſions found by *Leibnitz*'s method of differences may be proved to be true, notwithſtanding the error or inaccuracy of his principles: for ſuppoſing in his method that the mark of equality = does not ſignify truly but nearly equal in the ſenſe before given, and then correct your concluſion by the forementioned axiom, and you may be ſure it is juſt.

3. HE repreſents the Mathematicians as founding their reaſonings on maxims ſhocking to good ſenſe, as particularly when they take it for granted that a finite quantity divided by nothing is infinite, *Query* 16. I ſuppoſe our Author will not pretend to ſay here that he deſigned only to aſk a civil and innocent queſtion, but to charge the Mathematicians with proceeding on ſuch abſurd maxims as this; and if ſo, it is an haſty and ill-grounded charge; 'tis a

charge, I think, he himſelf can hardly imagine to be true. Does our Author take all Mathematicians to be fools; or did ever any fool imagine that a reaſonable anſwer could be given to this queſtion, How many nothings will fill a quart; and that the proper anſwer is, An infinite number? Yet this is only to ſay that a quart divided by nothing is infinite. 'Tis allowed that this is a rule in Algebra, *that finite divided by nothing is infinite:* but the meaning of it is only this, that if I inquire how big any quantity muſt be taken in order to anſwer any purpoſe, and the anſwer come out according to the rules of Algebra $\frac{1}{0}$ or $\frac{2}{0}$ *&c.* this is a ſign that no finite quantity is large enough. And is there any thing ſhocking in this to the ſenſe of any man, from the porter to the philoſopher? The ſame kind of anſwer ought to be given in relation to other maxims of Algebra, which appear ſtrange to thoſe unaccuſtomed to the phraſes, and really are in themſelves ſenſeleſs expreſſions, as that the ſum or product of two impoſſible quantities may be poſſible. Nay this may be ſaid of the terms multiply and divide, add and ſubtract, as uſed in Algebra. An Algebraiſt never ſcruples to ſubtract a greater quantity from a leſs; but if he really deſigns to do this, he may try till his heart akes before he will be able to accompliſh it, or to know what he is about. All theſe phraſes are therefore to be conſidered merely as terms of art to help the memory in an algebraic proceſs, or ſigns of rules, which from other principles we muſt know to be juſt, and not from any idea we have of the things, which, according to the common uſe of the words, an ignorant man may fancy they are intended to expreſs. 'Tis pity, indeed, theſe things have not been more particularly explained by the writers of Algebra, the want of which may well make it abſtruſe and confounding to beginners. Yet for a perſon to make theſe objections againſt the art or thoſe that underſtand it, is as if I ſhould impute it to the Logicians as a fault in theirs, that they uſe the horrid terms *Barbara*, *Celarent*, &c. Nor do I think the uſe of theſe abſurd expreſſions peculiar to Algebra, but they are frequently uſed where, thro' their being familiar, and the deſign of them eaſily intelligible, they are not thought to have any odd ſound. Thus it is a maxim in Arithmetic that $2 \times 0 = 5 \times 0$, or in general any number multiplied by 0 is $= 0$. Now I ſay that if by this any thing elſe be underſtood than that this is a good rule to go by in managing figures in Arithmetic, I will ſo far venture the laugh of the Public as to declare I don't underſtand it. He that can take two nothings and find their ſum, and then five nothings and find their ſum equal to the former, ſeems to me to be in a fair way to be able to divide by nothing, and find the quotient; and, with a little more pains, may prove that two is equal to five. To multiply by nothing, is as abſurd as to divide by nothing; and to ſuppoſe we can do either, is to imagine nothing to be a real quantity or number: for *nihili nullæ ſunt affectiones.* Here I ſuppoſe our Author agrees with me from his 40th *Query*; but no one will ſay that for this reaſon we ought to reject the maxim conſidered as a rule of Arithmetic, that the product of any number by o is = o.

4thly, *Query* 50. HE repreſents the diſputes and controverſies among Mathematicians as diſparaging the evidence of their methods: and, *Query* 51. he repreſents Logics and Metaphyſics as proper to open their eyes, and extricate them out of their difficulties. Now were ever two things thus put together? If the diſputes of the profeſſors of any ſcience diſparage the ſcience itſelf, Logics and Metaphyſics are much more diſparaged than Mathematics; why therefore, if I am half blind, muſt I take for my guide one that can't ſee at all? And to ſay the truth, it can hardly be look'd upon as fair to repreſent Mathematics as diſparaged by the diſputes of its profeſſors, which inſinuates that in this way it is peculiarly diſparaged, whereas the quite contrary is true, there being fewer diſputes among Mathematicians, as ſuch, than any other perſons whatſoever. The diſputes of Mathematicians about metaphyſical and philoſophical principles have nothing to do here; and take away theſe, hardly any remain.

Laſtly, HIS frequent inſinuations that Mathematicians don't care to be tried by the rules of good logic, and require indulgence for incorrect and falſe reaſonings, and think that the truth of the premiſes are proved without more ado by the truth of the concluſion, are entirely groundleſs. They, as well as all other mortals, deſire indulgence for inaccurate expreſſions, but none for falſe reaſoning; and for following the exacteſt rules of logic, they pride themſelves in being the moſt perfect patterns.

To make a thing plain as a propoſition in *Euclid*, is to give it the laſt degree of evidence. * Nor is it any objection to the juſtneſs of their reaſoning, that an algebraical note is ſometimes to be interpreted, at the end of the proceſs, in a ſenſe which cou'd not have been ſubſtituted in the beginning of it; ſince if quantities themſelves are conſidered as continually changing, the ſenſe of the mark which repreſents or expreſſes them muſt, in order to its doing ſo, continually change along with them. And they never imagine that any particular ſuppoſition can come under a general caſe which is inconſiſtent with the reaſoning thereof, or any juſt reaſoning whatſoever: tho' they have many times good reaſon to conclude that particular caſes in a general theorem are true, tho' they could not be proved in the ſame manner with the theorem itſelf.

To conclude; as I would not be thought, by any thing I have ſaid, to be an enemy to true Logic and ſound Metaphyſics; and on the contrary think the moſt general uſe of the Mathematics is to inure us to a juſt way of thinking and arguing; it is a proper inquiry, I imagine, for thoſe who have the direction of the education of youth, in what manner mathematical ſtudies may be ſo purſued as moſt ſurely to anſwer this end: upon which head the hints our Author gives, *Queries* 15, 38, 56, 57, deſerve to be conſider'd: for ſo

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* *Query* 43.

ſo far as Mathematics do not tend to make men more ſober and rational thinkers, wiſer and better men, they are only to be conſidered as an amuſement, which ought not to take us off from ſerious buſineſs.

FINIS.

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ERRATA.

PREFACE, pag. 5. l. 11. *for* a perſon *r.* perſons.
26. l. 23. *for* propoſitions *r.* proportions.
32. l. 16. *for* $2yx$ r. $2\dot{y}x$.

20

16 OCT 69